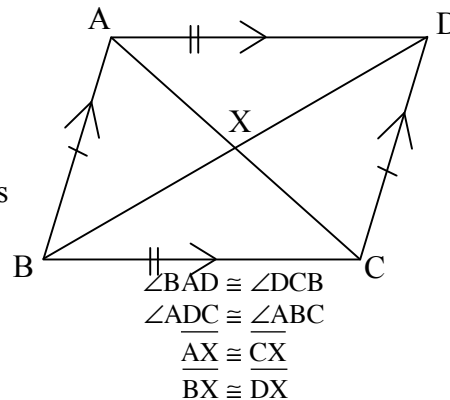


PROPERTIES OF QUADRILATERALS

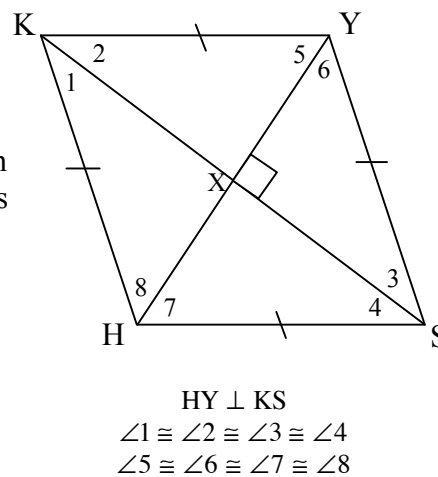
#13

Polygons with four sides are called **QUADRILATERALS**. There are various properties associated with specific kinds of quadrilaterals. The names of quadrilaterals studied in the book are listed below along with their properties.

Quadrilateral ABCD is a **parallelogram**. Opposite sides are parallel and congruent. Consecutive angles are supplementary. The diagonals bisect.

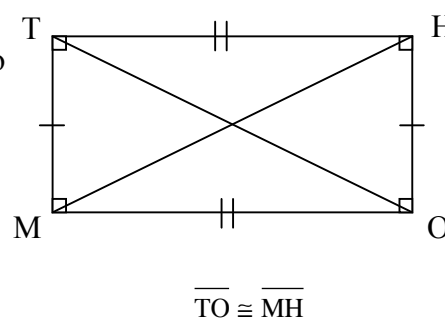


Quadrilateral KYSH is a **rhombus**. It is a parallelogram with four congruent sides. In addition to all the properties of a parallelogram, the diagonals are perpendicular and bisect the opposite angles.



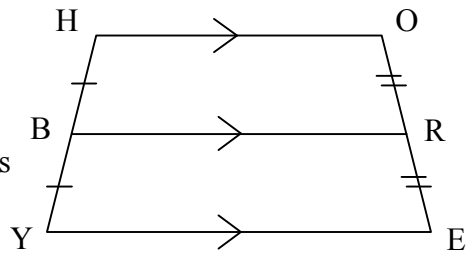
Quadrilateral THOM is a **rectangle**. It is a parallelogram with four right angles. In addition to all the properties of a parallelogram, the diagonals are congruent.

If a rectangle is also a rhombus, then it is called a **square**. A square would have all the properties listed previously.



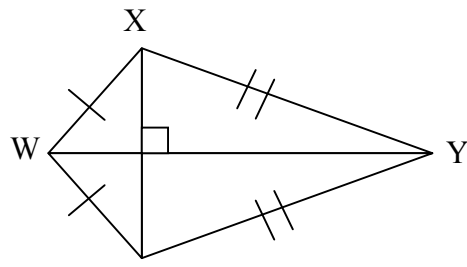
Two additional quadrilaterals are studied.

Quadrilateral HOEY is a **trapezoid**. It has exactly one set of parallel sides called bases. The segment connecting the midpoints of the non-parallel sides is parallel to the bases and has length equal to the average of the bases.



$$BR = \frac{1}{2}(HO + EY)$$

Quadrilateral WXYZ is a **kite**. It has one pair of opposite angles congruent and the diagonals are perpendicular.



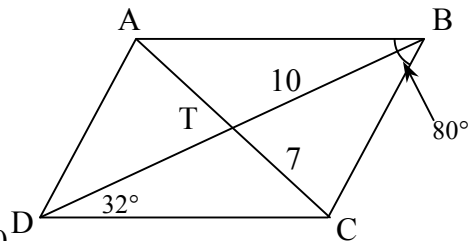
$$\begin{aligned} XZ &\perp WY \\ \angle WXY &\cong \angle WZY \end{aligned}$$

Example 1

Use the information in the parallelogram at right to find $m\angle BAD$, $m\angle ADB$, AT , and BD .

Solution: $m\angle BAD = 100^\circ$ since consecutive angles are supplementary.

$m\angle ADB = 80^\circ - 32^\circ = 48^\circ = 80^\circ - 32^\circ = 48^\circ$ since opposite angles are congruent. $AT = 7$ and $BD = 20$ since diagonals bisect.



Example 2

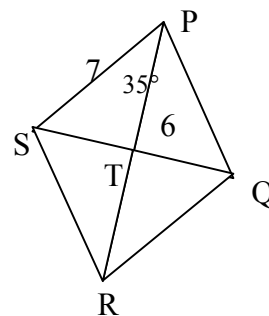
Use the information in the rhombus at right to find RQ , $m\angle SPQ$, $m\angle STR$, and RT .

Solution: $RQ = 7$ since all sides congruent.

$m\angle SPQ = 70^\circ$ since diagonals bisect opposite angles.

$m\angle STR = 90^\circ$ since diagonals are perpendicular.

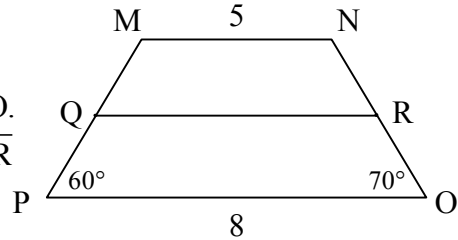
$RT = 6$ since diagonals bisect.



Example 3

Given that Q and R are midpoints in the trapezoid at right to find $m\angle QMN$, $m\angle QRN$, and QR.

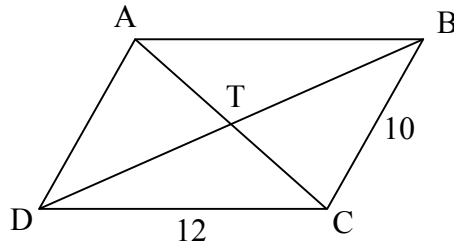
Solution: $m\angle QMN = 120^\circ$ since MN is parallel to PO.
 $m\angle QRN = 70^\circ$ since Q and R are midpoints, then \overline{QR} is parallel to the bases and with parallel lines, corresponding angles are congruent.
 $QR = \frac{1}{2}(5 + 8) = 6.5$.



Find the required information and justify your answers.

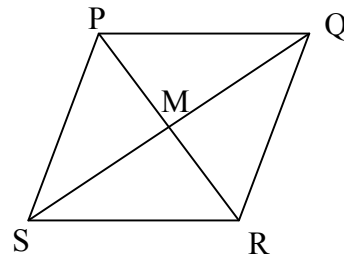
For problems 1-4 use the parallelogram at right.

1. Find the perimeter.
2. If $CT = 9$, find AT.
3. If $m\angle CDA = 60^\circ$, find $m\angle CBA$ and $m\angle BAD$.
4. If $AT = 4x - 7$ and $CT = -x + 13$, solve for x.



For problems 5-8 use the rhombus at right.

5. If $PS = \sqrt{6}$, what is the perimeter of PQRS?
6. If $PQ = 3x + 7$ and $QR = -x + 17$, solve for x.
7. If $m\angle PSM = 22^\circ$, find $m\angle RSM$ and $m\angle SPQ$.
8. If $m\angle PMQ = 4x - 5$, solve for x.

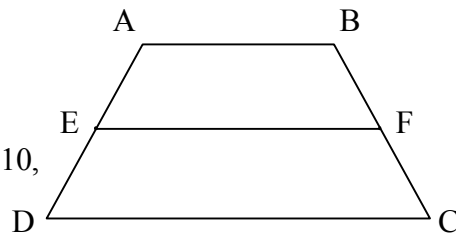


For problems 9-12 use the quadrilateral at right.

9. If $WX = YZ$ and $WZ = XY$, must WXYZ be a rectangle?
10. If $m\angle WZY = 90^\circ$, must WXYZ be a rectangle?
11. If the information in problems 9-10 are both true, must WXYZ be a rectangle?
12. If $WY = 15$ and $WZ = 9$, what is YZ and XZ?

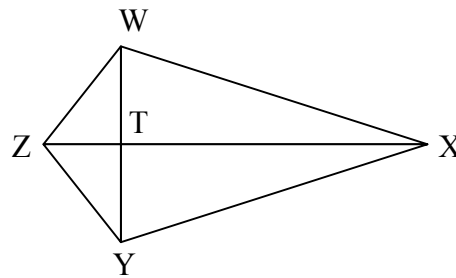


For problems 13-16 use the trapezoid at right with midpoints E and F.



13. If $m\angle EDC = 60^\circ$, find $m\angle AEF$.
14. If $m\angle DCB = 5x + 20$ and $m\angle ABC = 3x + 10$, solve for x .
15. If $AB = 6$ and $DC = 10$, find EF .
16. If $EF = 9$ and $DC = 15$, find AB .

For problems 17-20 use the kite at right.



17. If $m\angle XWZ = 95^\circ$, find $m\angle XYZ$.
18. If $m\angle WZT = 110^\circ$ and $m\angle WXY = 40^\circ$, find $m\angle ZWX$.
19. If $WZ = 5$ and $WT = 4$, find ZT .
20. If $WT = 4$, $TZ = 3$, and $TX = 10$, find the perimeter of $WXYZ$.

Answers

- | | | | |
|----------------|-----------------|--------------------------|------------------------|
| 1. 44 units | 2. 9 units | 3. $60^\circ, 120^\circ$ | 4. 4 |
| 5. $4\sqrt{6}$ | 6. 2.5 | 7. $22^\circ, 136^\circ$ | 8. 23.75 |
| 9. no | 10. no | 11. yes | 12. 12, 15 |
| 13. 60° | 14. 18.75 | 15. 8 | 16. 3 |
| 17. 95° | 18. 105° | 19. 3 | 20. $10 + 2\sqrt{116}$ |