Vector and Matrix Quantities   N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.

   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.

   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.